Lecture-2

Lecture for G&FI holography bootcamp.

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Outline :

1. Derive two point function of scalar primary operators from the bulk.
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2. Introduction to holographic RG.

The holographic prescription for the effective field theory action is :

$$Z[\tilde{\Phi}_k] = \left\langle e^{\int \tilde{\phi}_k O_k} \right\rangle = Z_{gravity} \Big|_{\phi_k \text{ at bndy} = \tilde{\phi}_k}.$$

/9802150 9k Witten

We work in the Poincaré patch

/9804058 Freedman, Mathur, Matusis, Rastelli

$$AdS_{d+1} \qquad ds^2 = \frac{1}{z^2}\left(dz^2 + \sum_{i=1}^{d} dx_i^2\right)$$

$$S = \frac{1}{2}\int d^dx\, dz \sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + m^2\phi^2\right)$$

$$\left(\phi(z, x) = \int [d^d\vec{k}]\, e^{i\vec{k}\cdot\vec{x}}\, \phi_{\vec{k}}(z) \right.$$

$$= \frac{1}{2}\int dz\, [d^d\vec{k}][d^d\vec{k}']\, \delta(\vec{k}+\vec{k}') \Big[z^{-d+1}\partial_z\phi_{\vec{k}'}\,\partial_z\phi_{\vec{k}} + z^{-d+1}k^2\phi_{\vec{k}'}\phi_{\vec{k}} + z^{-d-1}m^2\phi_{\vec{k}'}\phi_{\vec{k}} \Big]$$

e.o.m

$$\partial_z\left[z^{-d+1}\partial_z\phi_{\vec{k}}\right] = z^{-d+1}\vec{k}^2\phi_{\vec{k}} + z^{-d-1}m^2\phi_{\vec{k}}$$

put back sol

$$S_{on\text{-}shell} = \frac{1}{2}\int dz\,[d^dk]\left[\frac{1}{z^{d+1}}z^2\,\partial_z\phi_{-\vec{k}}\,\partial_z\phi_{\vec{k}} + \phi_{-\vec{k}}\,\partial_z\left(z^{-d+1}\partial_z\phi_{\vec{k}}\right)\right]$$

$$= \frac{1}{2}\int dz\,[d^dk]\left[\partial_z\left(\phi_{-\vec{k}}\,z^{-d+1}\partial_z\phi_{\vec{k}}\right)\right] \sim \lim_{z\to\epsilon} \frac{1}{2}\int [d^dk]\, z^{-d+1}\phi_{-\vec{k}}\,\partial_z\phi_{\vec{k}}$$

Thus the on-shell action is a total derivative and gets contribution from the boundary.

We now solve the e.o.m with bndy condition $\phi_k(z\to 0) = \tilde{\phi}_k$

Thus write: $\phi_k(z) = K^{\epsilon}(z,k)\,\tilde{\phi}_k$ — (2) and $\phi_k(z\to\infty) = 0$

such that $\lim_{z\to\epsilon} K^{\epsilon}(z,k) = 1$

and $\lim_{z\to\infty} K^{\epsilon}(z,k) = 0.$

If we find a solution of from (2) then

$$S_{\text{on-shell}} = \lim_{z\to\epsilon} \frac{1}{2}\int [d^d\bar{k}][d^d\bar{k}']\,\delta(\bar{k}+\bar{k}')\, z^{-d+1} K^{\epsilon}(z,\bar{k}')\,\tilde{\phi}_{\vec{k}'} \left(\partial_z K^{\epsilon}(z,k)\right)\tilde{\phi}_{\vec{k}}.$$

$$\langle O_{\vec{p}}\, O_{\vec{p}'}\rangle = \lim_{\epsilon\to 0} \frac{\delta}{\delta\tilde{\phi}_{\vec{p}}}\frac{\delta}{\delta\tilde{\phi}_{\vec{p}'}} e^{-S_{\text{on-shell}}}$$

$$= -\lim_{\epsilon\to 0}\lim_{z\to\epsilon} \delta(\vec{p}+\vec{p}')\, z^{-d+1} K^{\epsilon}(z,\vec{p}')\left(\partial_z K^{\epsilon}(z,\vec{p})\right)$$

$$= -\lim_{\epsilon\to 0} \epsilon^{-d+1}\,\delta(p+p')\,\left|\partial_z K^{\epsilon}(z,\vec{p})\right|_{z=\epsilon}$$

So we will be done when we find the scalar solution.

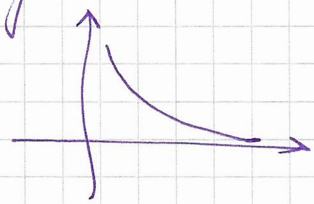
e.o.m

$$(1-d)\, z^{-d}\, \partial_z \phi_k + z^{-d+1}\, \partial_z^2 \phi_k = z^{-d+1} k^2 \phi_k + z^{-d-1} m^2 \phi_k$$

$$z^2 \partial_z^2 \phi_k + (1-d)\, z\, \partial_z \phi_k - \left(z^2 k^2 + m^2\right) \phi_k = 0.$$

Plugging into mathematica give J_ν and Y_ν.

Better to write as K_ν



$$z^2 \frac{d^2 K_\nu(z)}{dz^2} + z \frac{d K_\nu(z)}{dz} - \left(z^2 + \nu^2\right) w = 0$$

Solving:

$$\phi_k(z) = \frac{z^{d/2} K_\nu(\bar{k} z)}{\epsilon^{d/2} K_\nu(\bar{k}\epsilon)} \tilde{\phi}_{\bar{k}} \qquad \nu = \sqrt{\frac{d^2}{4} + m^2}$$

$$K_\epsilon(\vec{k}, z) = \left(\frac{z}{\epsilon}\right)^{d/2} \frac{K_\nu(\bar{k} z)}{K_\nu(\bar{k}\epsilon)}.$$

$$\therefore \langle O_{\bar{k}} O_{\bar{k}'} \rangle = -\delta(\bar{k} + \bar{k}') \lim_{\epsilon \to 0} \epsilon^{-d+1} \partial_z K_\epsilon(z, \vec{p}) \Big|_{z=\epsilon}$$

Can use:

$$K_\nu(kz) = 2^{\nu-1} \Gamma(\nu) (kz)^{-\nu} (1 + \dots) - 2^{-\nu-1} \frac{\Gamma(1-\nu)}{\nu} (kz)^{\nu} (1 + \dots)$$

$$= -\delta(\vec{k} + \vec{k}') \lim_{\epsilon \to 0} \epsilon^{-d+1} \partial_z \frac{2^{\nu-1} \Gamma(\nu) (kz)^{-\nu+\frac{d}{2}} (1 + \dots) - 2^{-\nu-1} \frac{\Gamma(1-\nu)}{\nu} (kz)^{\nu+\frac{d}{2}} (1 + \dots)}{(\epsilon k)^{-d/2} \, (k\epsilon)^{-\nu} \Gamma(\nu)\, 2^{\nu-1} \dots - 2^{-\nu-1} \frac{\Gamma(1-\nu)}{\nu} (k\epsilon)^{\nu} \dots}$$

$$= -\frac{2^{-2\nu}\,\Gamma(1-\nu)}{\Gamma(1+\nu)}\, k^{-2\nu}\, \epsilon^{2\nu-d}\,(2\nu)\,\delta(\vec{k}+\vec{k}') + \cdots$$

$$\Delta = \frac{d}{2} + \nu$$

$\Big\downarrow$ Fourier transform.

$$\langle O(x)\, O(y)\rangle = \frac{1}{\pi^{d/2}}\, \epsilon^{2(\Delta-d)} \left(\frac{2\Delta-d}{\Delta}\right) \frac{\Gamma(\Delta+1)}{\Gamma(\Delta-\frac{d}{2})}\, |\vec{x}-\vec{y}|^{-2\Delta}$$

But the bulk also describes $O(x)$ with conformal dimension $= d-\Delta$.

If we take:

$$K_\epsilon(\vec{k}, z) = \left(\frac{z}{\epsilon}\right)^{d/2} \frac{K_\nu(\vec{k}z)}{2^{\nu-1}\,\Gamma(\nu)\,(k\epsilon)^{-\nu}}$$

Sample calculations

$$\lim_{\epsilon\to 0} \left(\partial_z K_\epsilon(\vec{k}, z)\right)\Big|_{z=\epsilon} = \Big[\, 2^{\nu-1}\,\Gamma(\nu)\, k^{-\nu+\frac{d}{2}} \left(-\nu+\frac{d}{2}\right) \epsilon^{-\nu+\frac{d}{2}-1}$$

$$- 2^{-\nu-1}\frac{\Gamma(1-\nu)}{\nu}\, k^{\nu+\frac{d}{2}} \left(\nu+\frac{d}{2}\right) \epsilon^{\nu+\frac{d}{2}-1}$$

$$k^{-\frac{d}{2}}\, \epsilon^{d}\, \epsilon^{1-\frac{d}{2}} \qquad \overline{\qquad 2^{\nu-1}\,\Gamma(\nu)\,(k\epsilon)^{-\nu} \qquad}$$

$$= -\, \epsilon^{d}\, k^{2\nu}\, \frac{\Gamma(1-\nu)}{\nu\,\Gamma(\nu)}\, 2^{2\nu}\left(\nu+\frac{d}{2}\right)$$

Holographic RG:

Double trace deformation

large N CFTs.

$$e^{-S_0 - f_0 \int \mathcal{O}^2}$$

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle = \langle \mathcal{O}\mathcal{O}\rangle \langle \mathcal{O}\mathcal{O}\rangle \cdots + \text{permutations}$$

$$\langle \mathcal{O}\mathcal{O}\rangle_f = \int \mathcal{D}\phi \; \mathcal{O}\mathcal{O} e^{-S_0} \left(1 - \frac{f_0}{2}\int \mathcal{O}^2 \cdots\right)$$

$$= G_0 - f_0 G_0^2 + \cdots$$

$$= \frac{G_0}{1 + f_0 G_0} = G_0 - f_k G_0^2$$

(Schwinger Dyson).

$$f_k = \frac{f_0}{1 + f_0 G_0(k)}$$

$$\int d^d x \, f \, \underbrace{\mathcal{O}_2}_{d-2\nu}$$

We want $\Delta = \Delta_-$ because we want relevant deformation

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$$[\mathcal{O}^2] \cancel{= + 2\Delta_+ = d + 2\nu} = 2\Delta_- = (d - 2\nu)$$

$$\therefore [f] = 2\nu.$$

$$\therefore f = \lambda(\mu)\, \mu^{2\nu}$$

$$G_0(k) = b\, k^{-2\nu}$$

$$G_0 = b k^{-2\nu} \qquad \text{dim of } \mathcal{O} : \Delta_- = \frac{d}{2} - \nu$$

$$\therefore \left[\mathcal{O}^2\right] = d - 2\left(\frac{d}{2} - \nu\right)$$

$$= 2\nu > 0 \quad \therefore \underline{\text{Relevant!}}$$

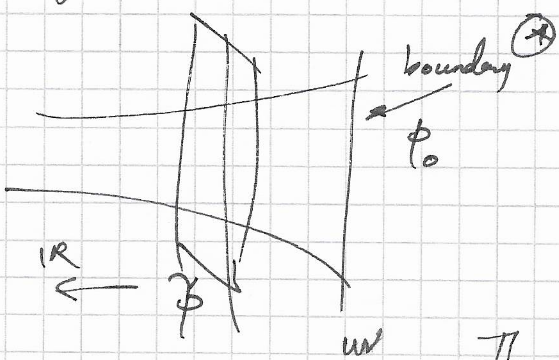
$$\Rightarrow \quad \lambda \mu^{2\nu} = \frac{f_0}{1 + f_0 b \mu^{-2\nu}}$$

$$\therefore \quad \lambda = \frac{f_0}{\mu^{2\nu} + f_0 b} \Rightarrow f_0 = \frac{\lambda \mu^{2\nu}}{1 - b\lambda}$$

$$\Rightarrow \quad \mu \frac{d\lambda}{d\mu} = -2\nu\lambda + 2\nu b \lambda^2$$

Reference <u>arXiv 1010.1264v2</u> Heemskerk & Polchinski

(*) Note in lecture used χ for $\tilde{\phi}$ and $\tilde{\phi}$ for ϕ_0.



$$\frac{dz^2 + dx_\mu dx_\nu \eta^{\mu\nu}}{z^2}$$

has isometry : $x^\mu \to \lambda x^\mu$

$z \to \lambda z$

Thus going into bulk is going to IR of the theory.

$$Z = \int \mathcal{D}\tilde{\phi} \, \Psi_{IR} \Psi_{UV} \quad \text{where} \quad \Psi_{UV} = \int \mathcal{D}\phi \, e^{-S[\phi]} \leftarrow \int_{z=\epsilon}^{z=l} dz \ldots \text{ with } \phi(z=\epsilon) = \phi_0, \ \phi(z=l) = \tilde{\phi}.$$

$$\Psi_{IR} = \int \mathcal{D}\phi \, e^{-S} \leftarrow \int_l^\infty dz \ldots \text{ with } \phi(z=l) = \tilde{\phi}.$$